



# **From Statistical Inference to Probabilistic Reasoning**

**Using Bayesian Networks for Reasoning with Small Samples, Missing Values, and No Data.**



Hello  
my name is

Stefan  
Conrady

# The BayesiaLab Software Platform

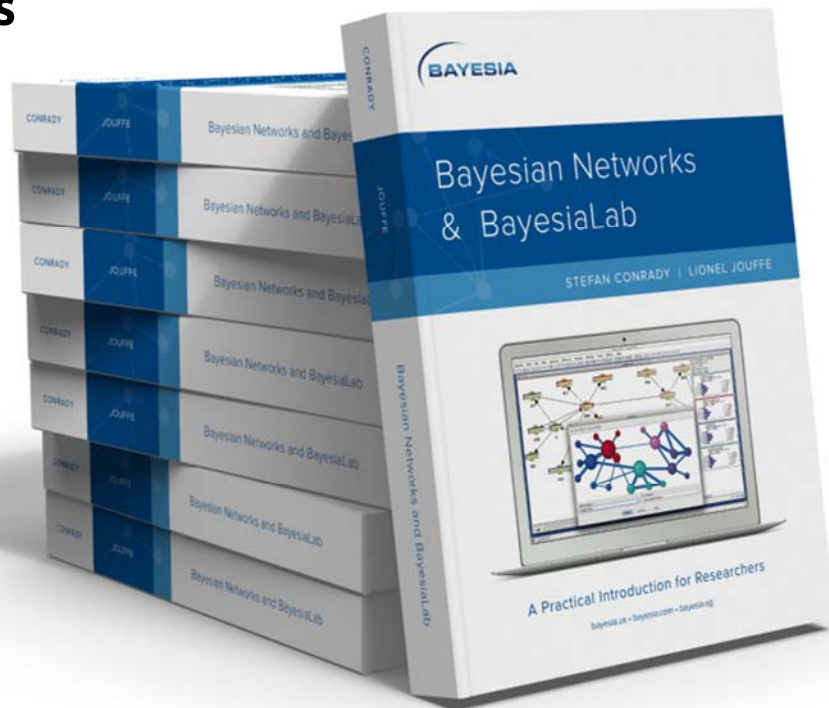




# Bayesian Networks & BayesiaLab

## A Practical Introduction for Researchers

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RUSSELL GLASS · SEAN CALLAHAN

THE

**BIG DATA  
DRIVEN**

**BUSINESS**

DATA  
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Decision-Making

O'REILL

**Data  
Driven**

Creating a Data Culture

5 Steps To Powering  
Data Driven Decision Making

increasing sales with  
**DATA - DRIVEN  
MARKETING**



loginradius

**DATA-DRIVEN** decisions in a  
**FORTUNE 500**

Data  
driven  
decisions

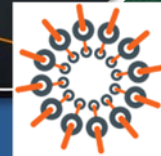


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# The End of Theory?





What if I have  
little or no  
data?

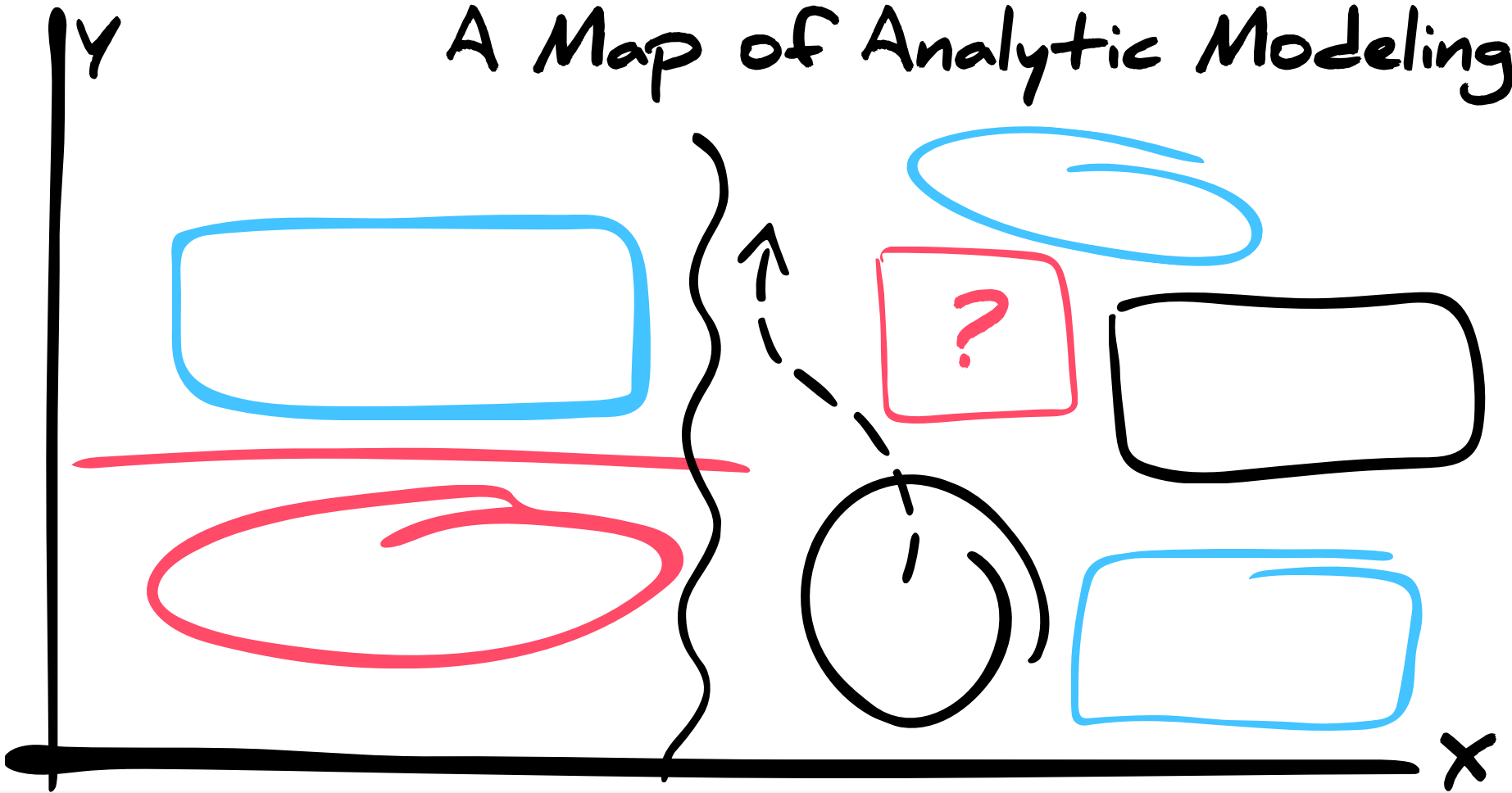


# Small Data in a Big-Data World

## Small-Data Challenges

- Generating knowledge from “small data”
  - Overparameterization
  - Variable selection
- Applying knowledge to “small data”
  - Incomplete observations
  - Uncertain observations
  - Hypothetical scenarios
  - Cost of observations

# A Map of Analytic Modeling



# The Purpose of Models

*Statistical Science*  
2010, Vol. 25, No. 3, 289–310  
DOI: 10.1214/10-STS330  
© Institute of Mathematical Statistics, 2010

## To Explain or to Predict?

Galit Shmueli

*Abstract* — Statistical modeling is a powerful tool for developing and testing hypotheses. It is used in a wide range of scientific disciplines for the purpose of theory building and testing, one finds a range of perceptions regarding the relationship between causal explanation and empirical prediction. In many scientific fields such as economics, psychology, education, and environmental science, statistical models are used almost exclusively for the purpose of prediction. This article focuses on the use of statistical modeling for causal explanation and for prediction. My main premise is that the two are often conflated, yet the causal versus predictive distinction has a large impact on each step of the statistical modeling process and on its consequences. Although not explicitly stated in the statistics methodology literature, applied statisticians instinctively sense that predicting and explaining are different. This article discusses a predictive goal. The purpose of this article is to clarify the distinction between explanatory and predictive modeling, to discuss its sources, and to reveal the practical implications of the distinction to each step in the modeling process.

*Key words and phrases:* Explanatory modeling, causality, predictive modeling, predictive power, statistical strategy, data mining, scientific research.

### 1. INTRODUCTION

Looking at how statistical models are used in different scientific disciplines for the purpose of theory building and testing, one finds a range of perceptions regarding the relationship between causal explanation and empirical prediction. In many scientific fields such as economics, psychology, education, and environmental science, statistical models are used almost exclusively for the purpose of prediction. This article focuses on the use of statistical modeling for causal explanation and for prediction. My main premise is that the two are often conflated, yet the causal versus predictive distinction has a large impact on each step of the statistical modeling process and on its consequences. Although not explicitly stated in the statistics methodology literature, applied statisticians instinctively sense that predicting and explaining are different. This article

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Description

Prediction

Explanation

Simulation

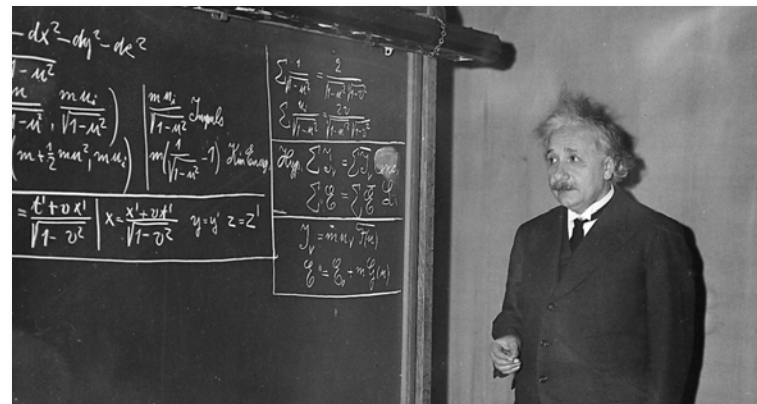
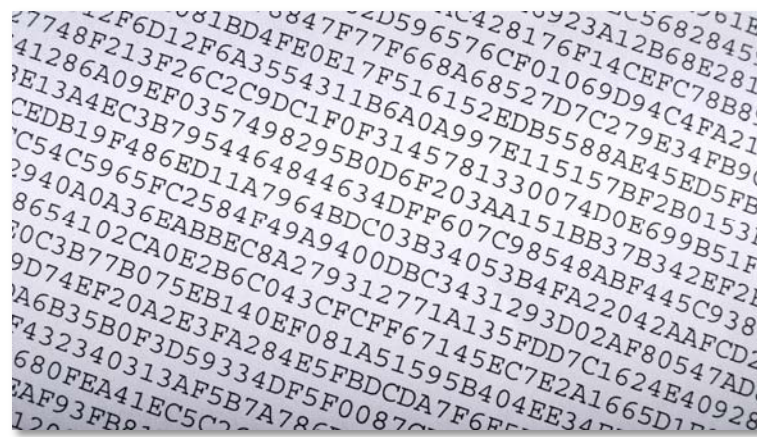
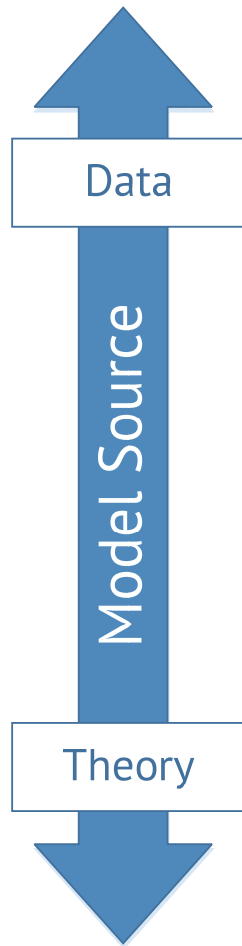
Optimization

Model Purpose

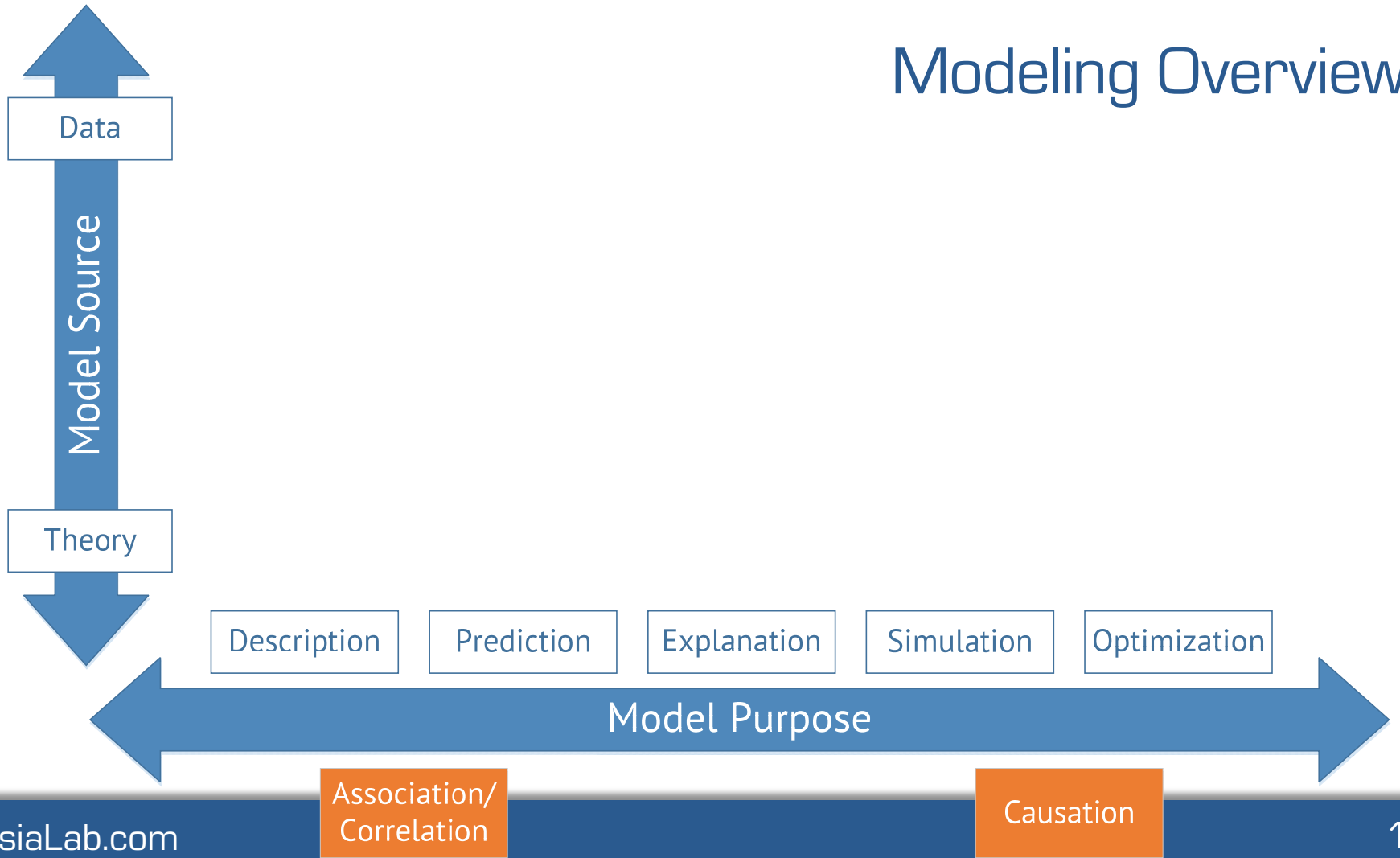
Association/  
Correlation

Causation

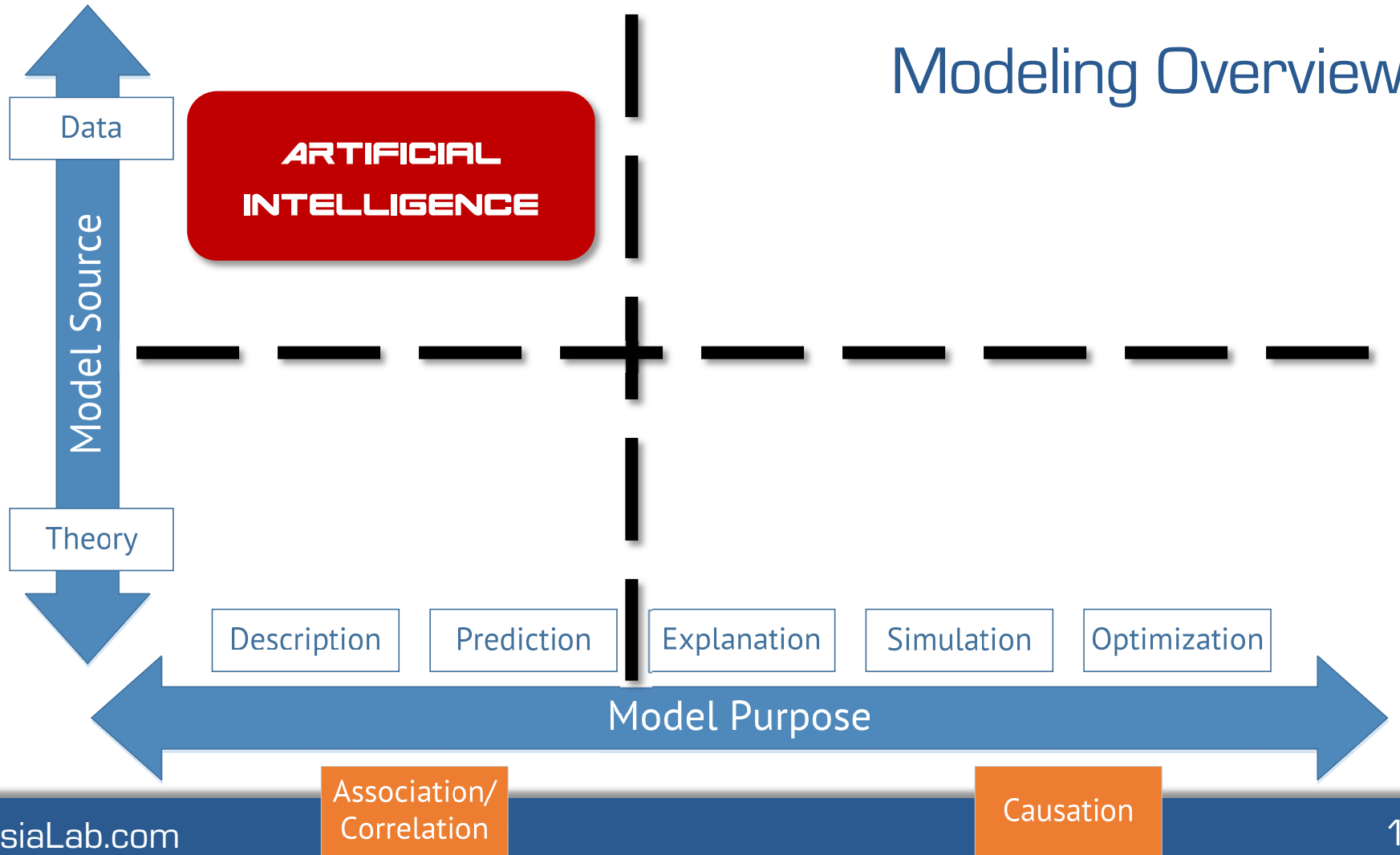
# Source of Models



# Modeling Overview



# Modeling Overview



# Bayesian Networks

Data

Model Source

Theory

Description

Prediction

Explanation

Simulation

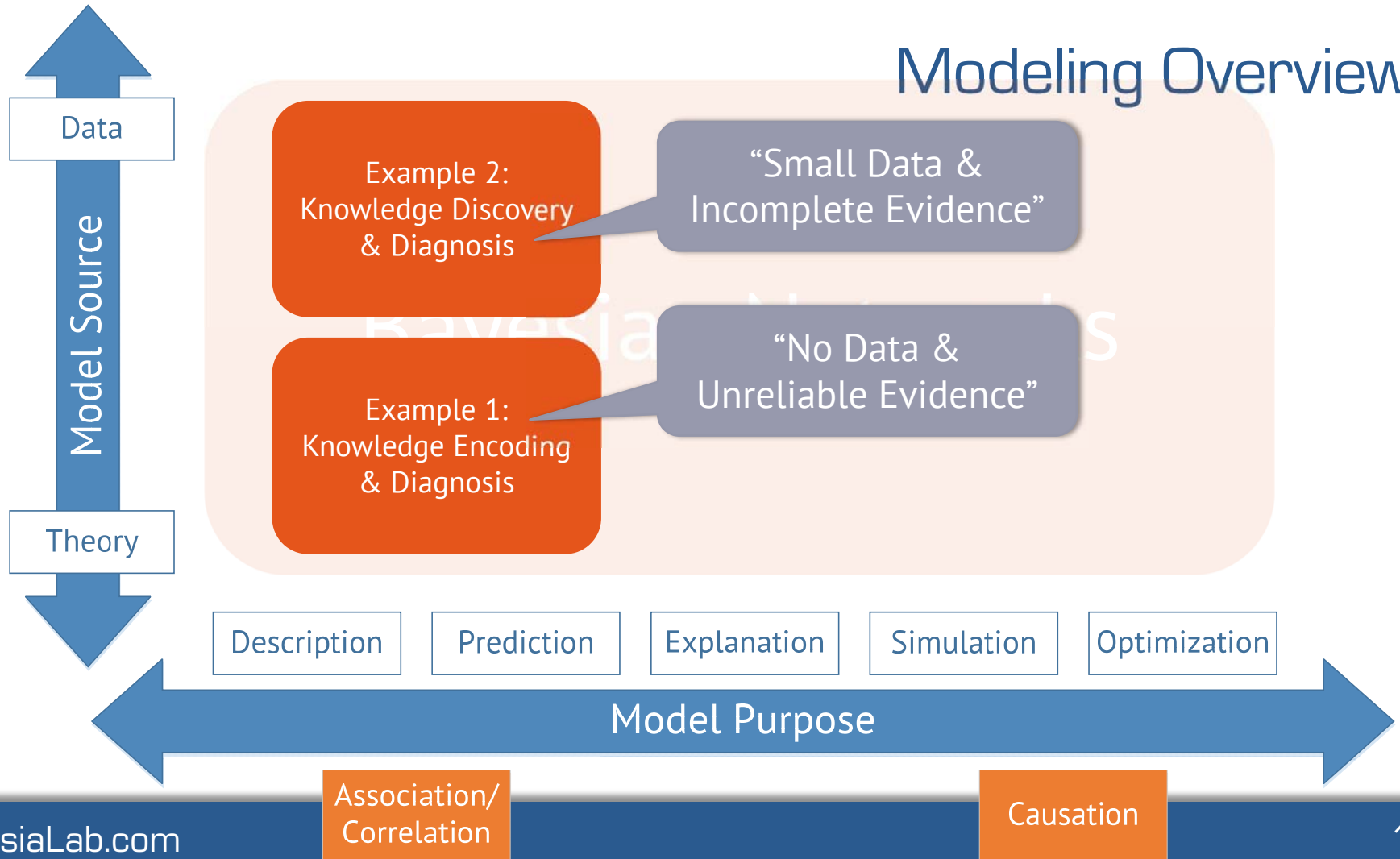
Optimization

Model Purpose

Association/  
Correlation

Causation

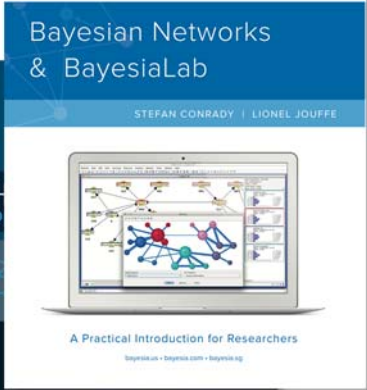
# Modeling Overview





# The New Paradigm: Bayesian Networks

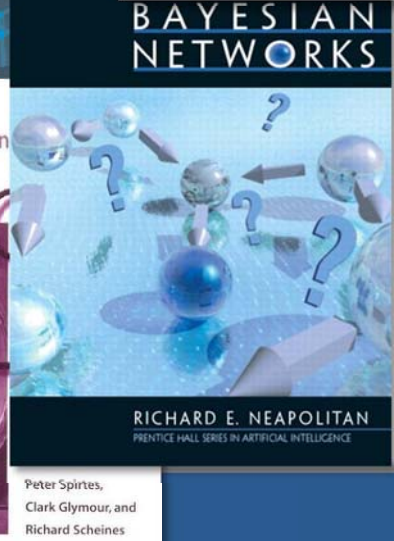
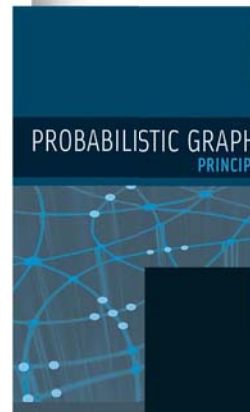
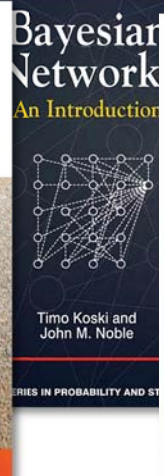
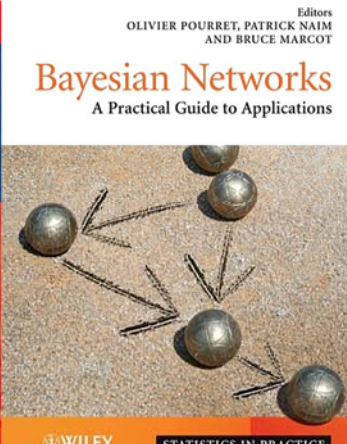
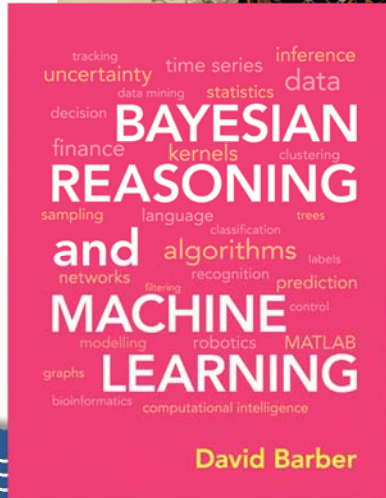
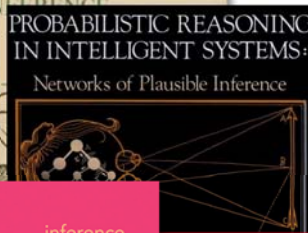
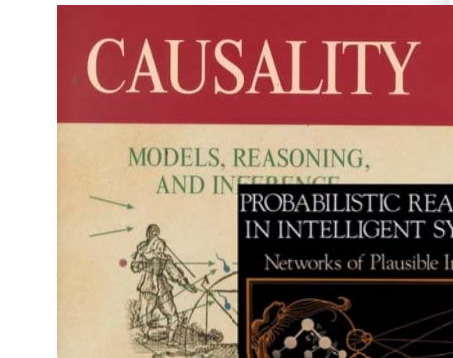
BAYESIA



## BAYESIAN NETWORKS\*

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University of California, Los Angeles, CA 90024  
[judea@cs.ucla.edu](mailto:judea@cs.ucla.edu)

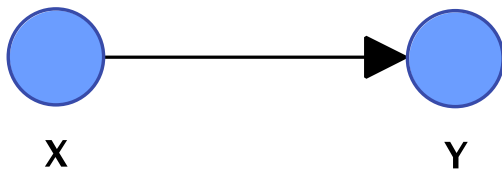
Bayesian networks were developed in the late 1970's to model distributed processing in reading comprehension, where both semantical expertise and statistical knowledge can be combined to form a coherent interpretation. The networks have since been used to fill a void in expert systems technology.



# Rev. Thomas Bayes

## Bayes's Theorem for Conditional Probabilities

$$P(X | Y) = \frac{P(Y | X)P(X)}{P(Y)}$$



T. Bayes.

## PHILOSOPHICAL TRANSACTIONS:

[ 37° ]

quodque solum, certa nitri signa præbere, sed plura concurrere debere, ut de vero nitro producto dubium non relinquatur.

LII. *An Essay towards solving a Problem in the Doctrine of Chances.* By the late Rev. Mr. Bayes, F. R. S. communicated by Mr. Price, in a Letter to John Canton, A. M. F. R. S.

Dear Sir,

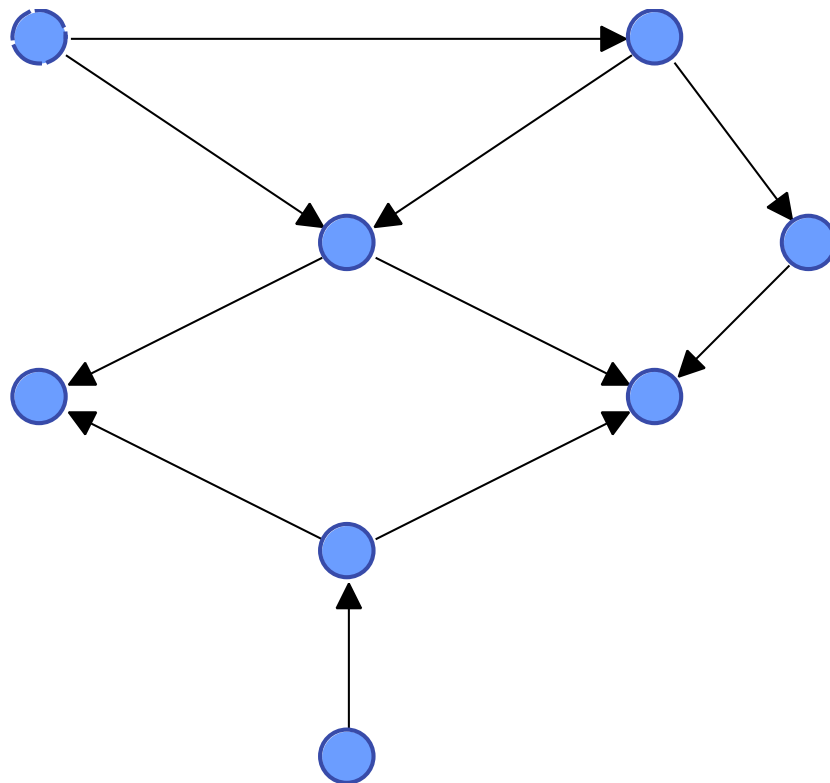
Read Dec. 23, 1763. I Now send you an essay which I have found among the papers of our deceased friend Mr. Bayes, and which, in my opinion, has great merit, and well deserves to be preserved. Experimental philosophy, you will find, is nearly interested in the subject of it; and on this account there seems to be particular reason for thinking that a communication of it to the Royal Society cannot be improper.

He had, you know, the honour of being a member of that illustrious Society, and was much esteemed by many in it as a very able mathematician. In an introduction which he has writ to this Essay, he says, that his design at first in thinking on the subject of it was, to find out a method by which we might judge concerning the probability that an event has to happen, in given circumstances, upon supposition that we know nothing concerning it but that, under the same circum-

# The New Paradigm: Bayesian Networks



## Probabilistic Graphical Model

- The graph *is* the model
- No formulas, no equations!



# The New Paradigm: Bayesian Networks

## Two Components:

- Node 
- Arc 

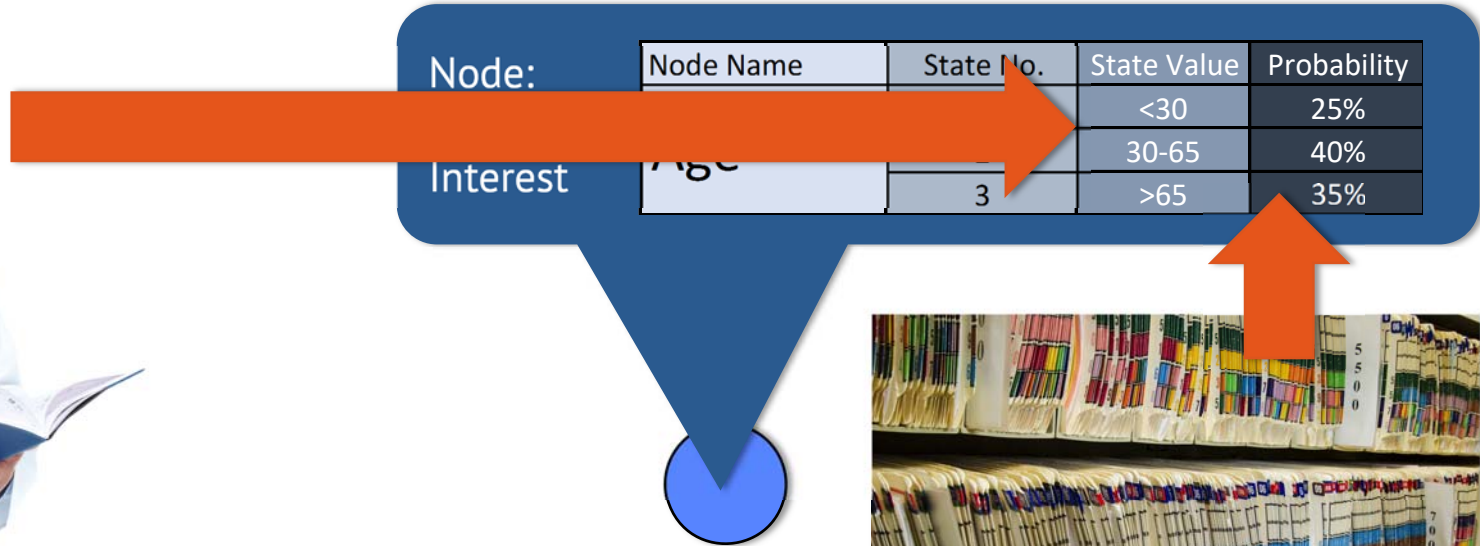
# The New Paradigm: Bayesian Networks

## Example

- A specialist in respiratory medicine summarizes his knowledge about his patients.



# The New Paradigm: Bayesian Networks



# The New Paradigm: Bayesian Networks



Node:  
Variable of  
Interest

Node Name	State No.	State Value	Probability
Smoker	1	TRUE	53.75%
	2	FALSE	46.25%

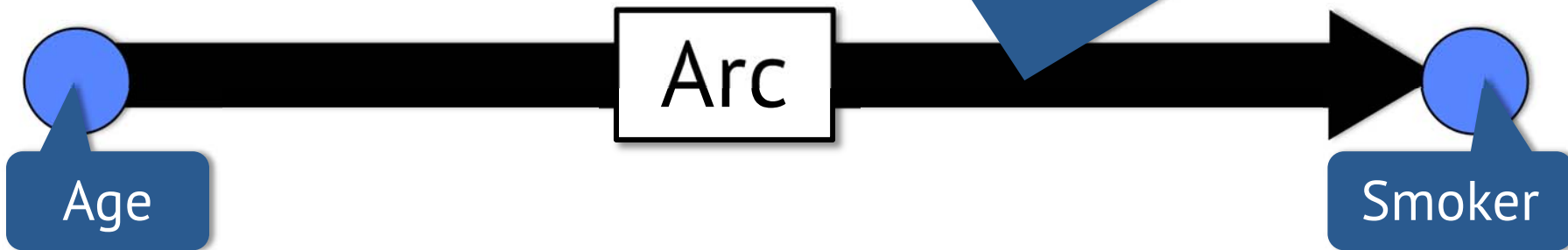
Age



# The New Paradigm: Bayesian Networks

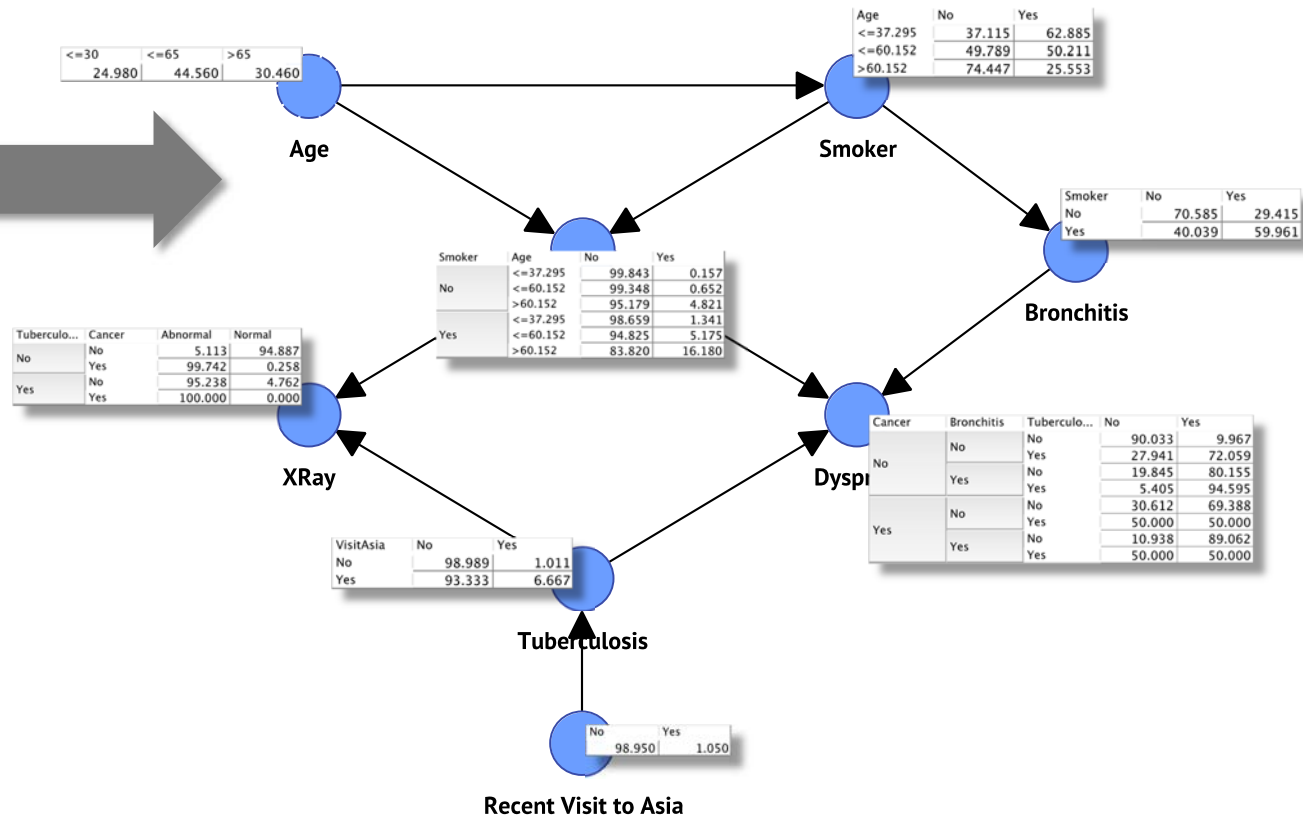
Discrete & Nonparametric  
Probabilistic Relationship  
 $P(Y|X)$

Age	Smoker	
	FALSE	TRUE
<30	31%	69%
30-65	53%	47%
>65	74%	26%





# The New Paradigm: Bayesian Networks



# The New Paradigm: Bayesian Networks

## Key Properties of Bayesian Networks

- Representation (or approximation) of the joint probability distribution of all variables.
- Numerical and categorical variables are treated identically.
- No distinction between dependent and independent variables.
- Nonparametric.

Compare to algebraic formula:

Representation of **one** variable of the joint probability distribution, i.e.  $y=f(x)$

Dependent

$$y = \beta_0 + \beta_1 x_1 + \dots + \beta_n x_n$$

Independent

Independent

# The New Paradigm: Bayesian Networks

## Key Properties of Bayesian Networks

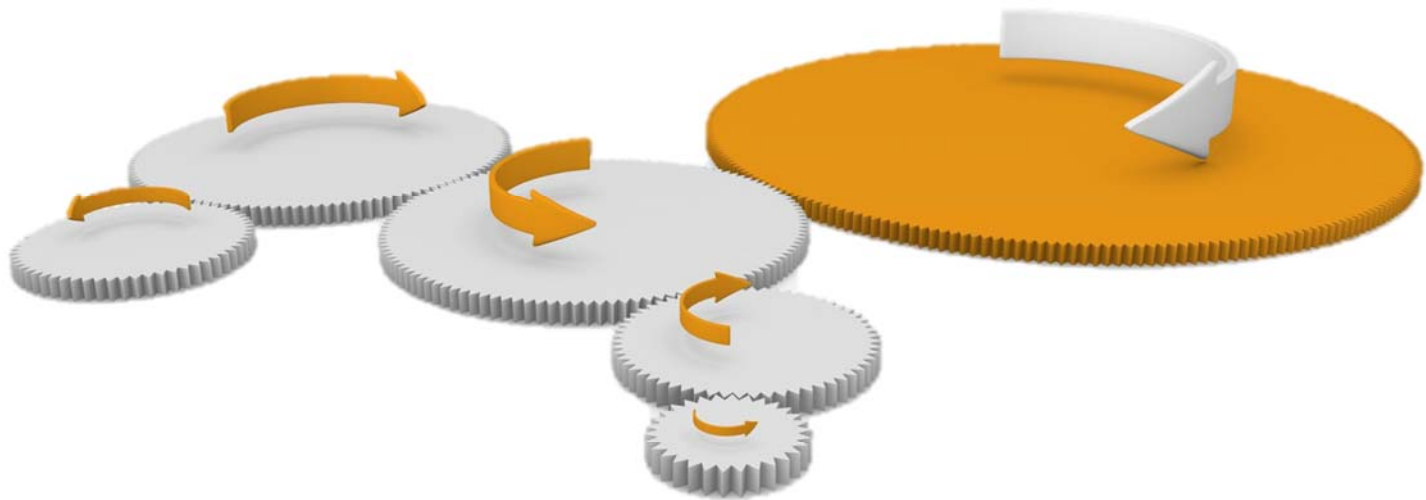
- Omni-directional Inference, i.e. evaluation is always performed in all directions.

Compare to “uni-directional” algebraic formula and human intuition

$$y \overset{\begin{array}{|c|} \hline \text{ONE} \\ \text{WAY} \\ \hline \leftarrow \\ \hline \end{array}}{=} \beta_0 + \beta_1 x_1 + \dots + \beta_n x_n$$

# The New Paradigm: Bayesian Networks

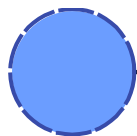
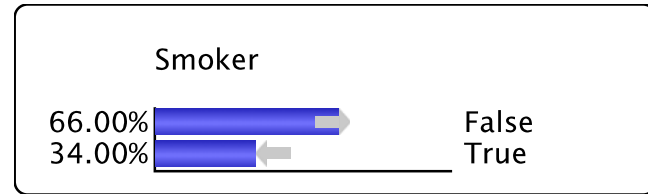
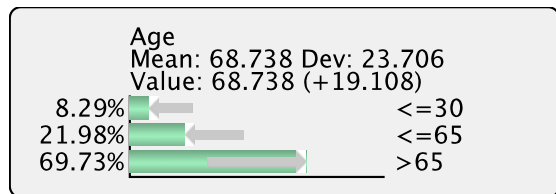
## Omni-Directional Inference



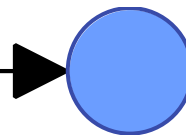
# The New Paradigm: Bayesian Networks

## Key Properties of Bayesian Networks

- Bayesian networks are inherently probabilistic.
- Evidence and inference are represented as distributions.
- Inference can be performed with partial evidence.



Age



Smoker

# The New Paradigm: Bayesian Networks

## Key Properties of Bayesian Networks

- Bayesian networks are inherently probabilistic.
- Evidence and inference are represented by distributions.
- Inference can be performed with partial evidence.

Deterministic  
Point Estimate

Compare to algebra

Single  
Value Input

Single  
Value Input

$$y = \beta_0 + \beta_1 x_1 + \dots + \beta_n x_n$$

# The New Paradigm: Bayesian Networks

## Key Properties of Bayesian Networks

- Bayesian networks can encode causal direction, algebra cannot.

### Limitations of Algebra

$$a = \frac{F}{m}$$

Causal Interpretation Possible

$$m = \frac{F}{a}$$

Causal Interpretation **Not** Possible

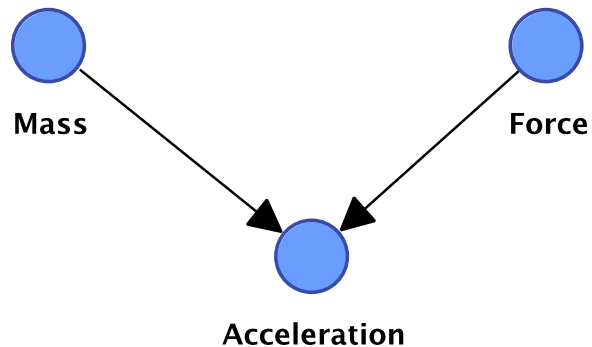
# The New Paradigm: Bayesian Networks

## Key Properties of Bayesian Networks

- Bayesian networks can encode causal direction, algebra cannot.

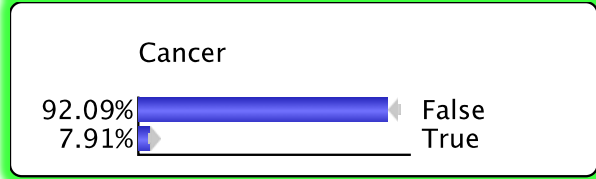
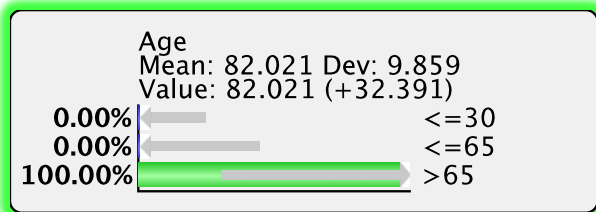
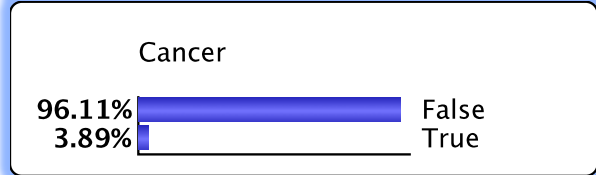
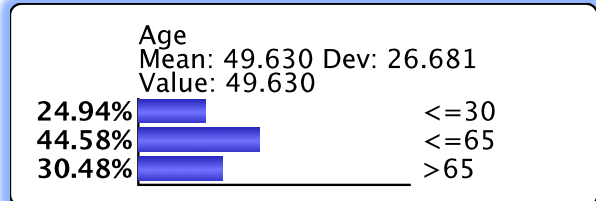
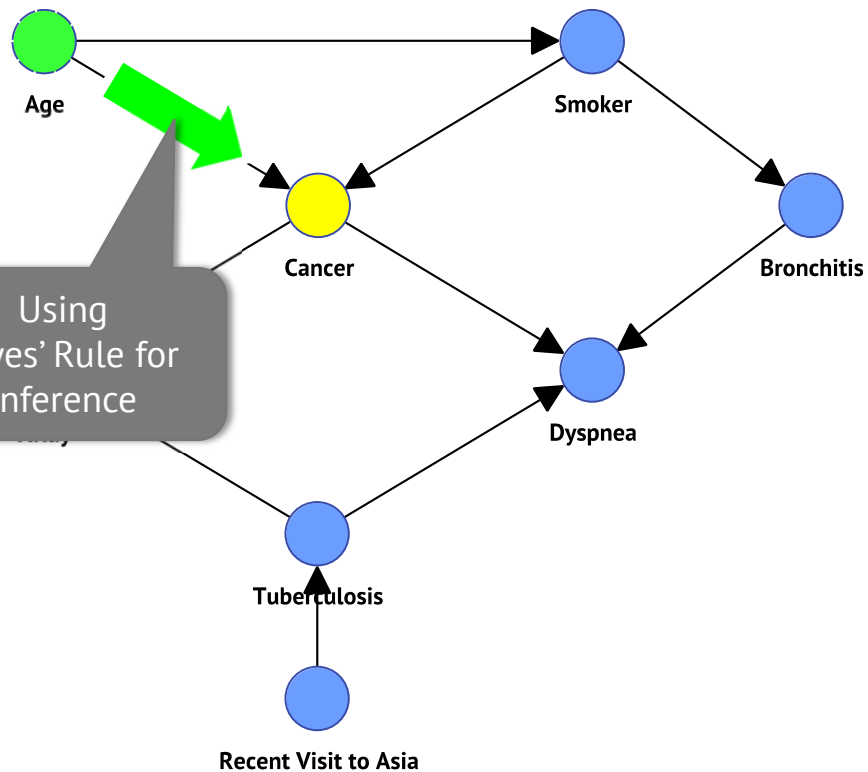
### Algebra vs. Bayesian Network

$$a = \frac{F}{m}$$

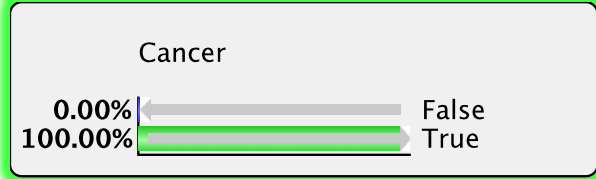
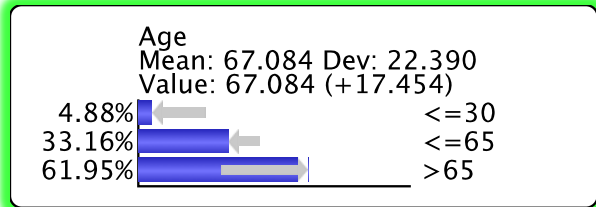
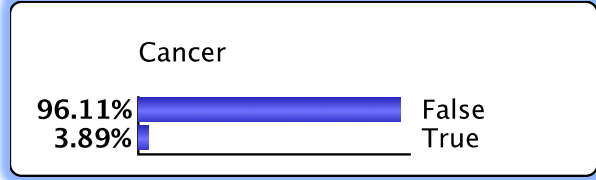
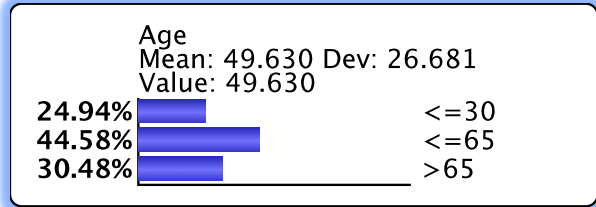
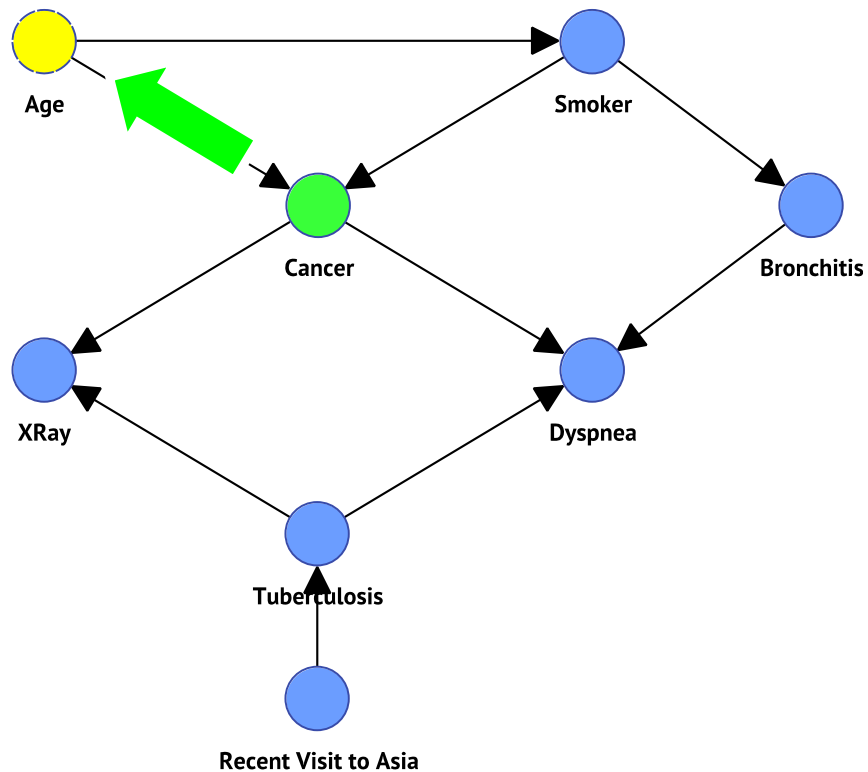




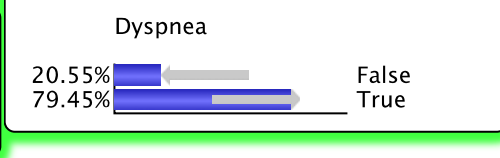
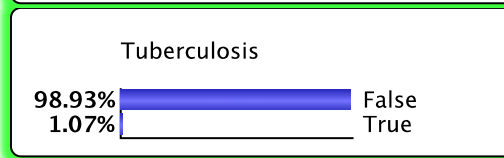
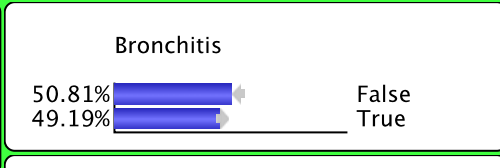
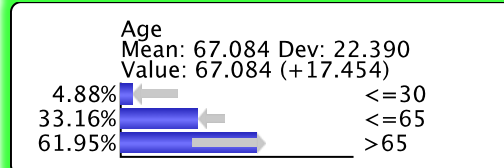
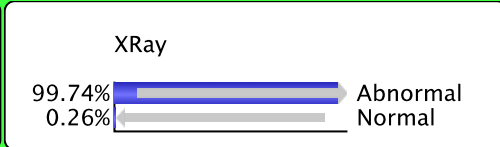
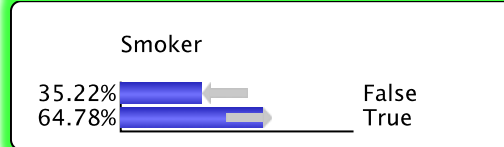
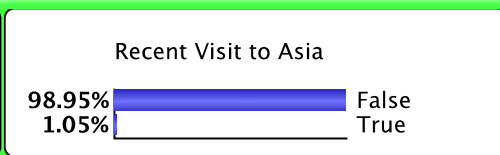
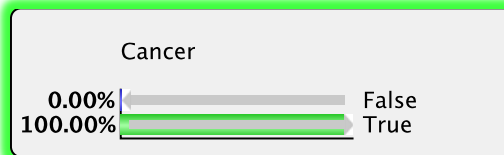
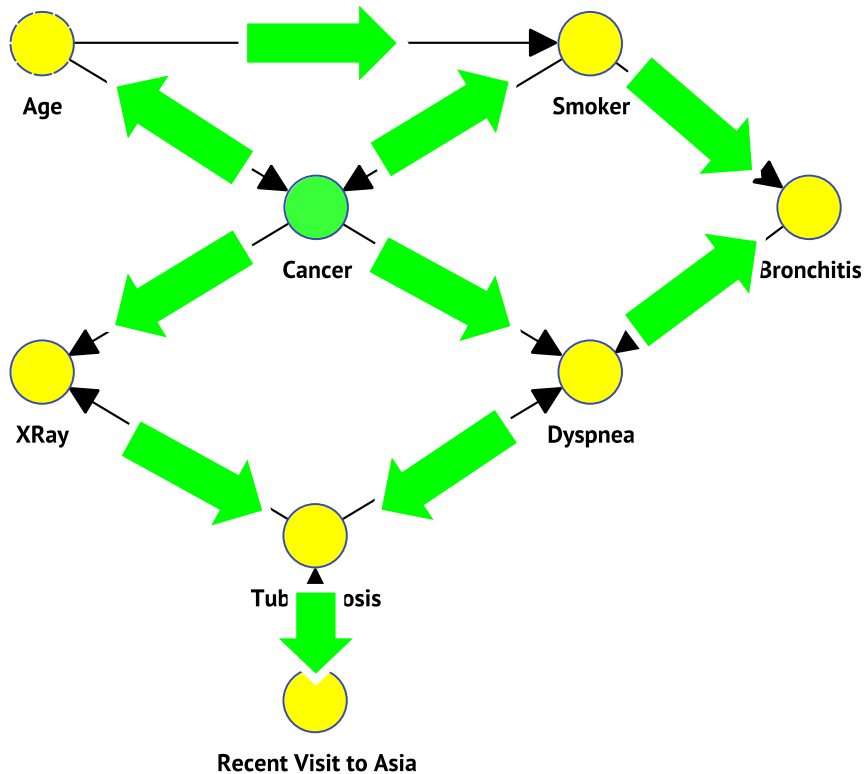
# The New Paradigm: Bayesian Networks



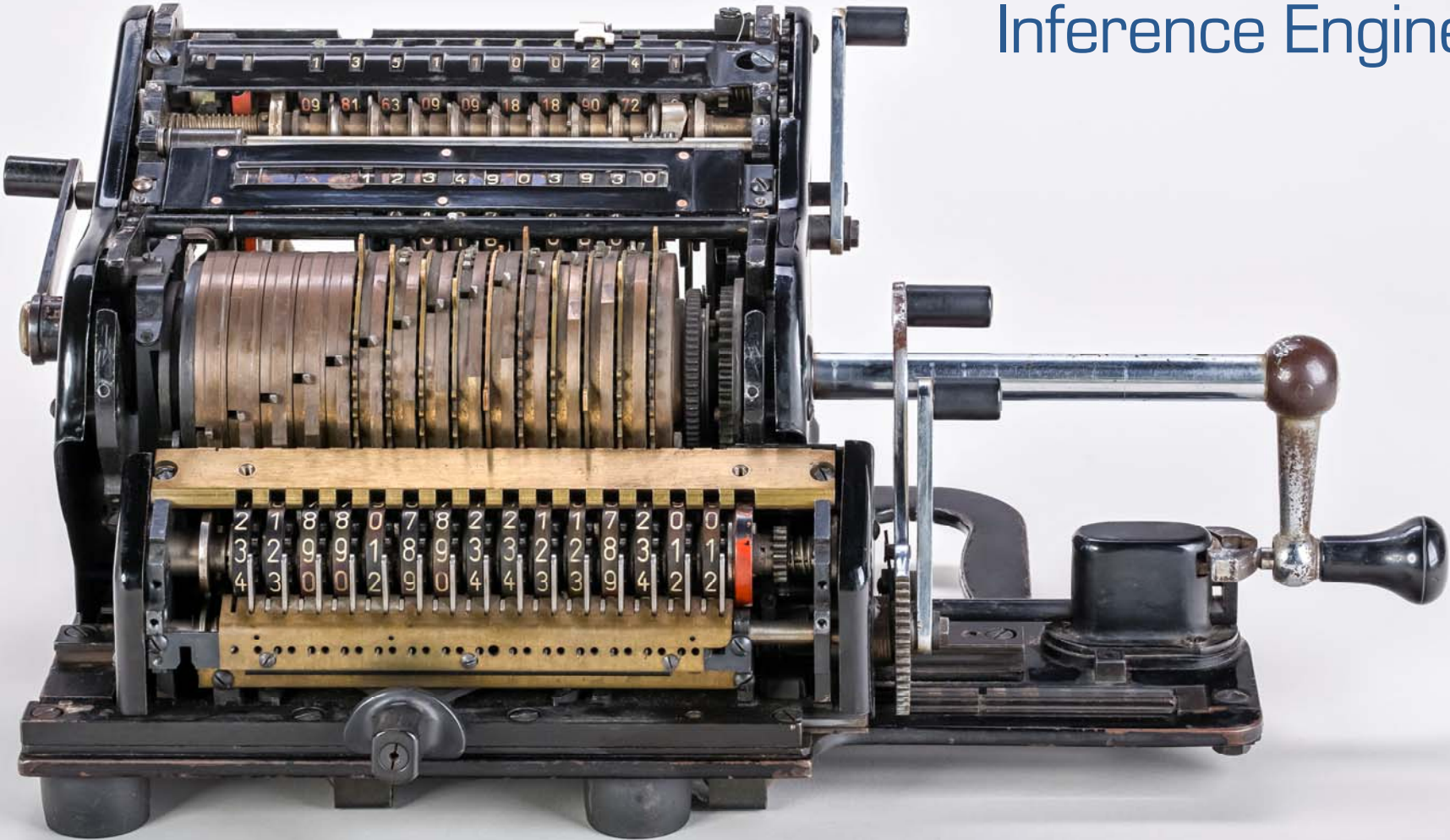
# The New Paradigm: Bayesian Networks



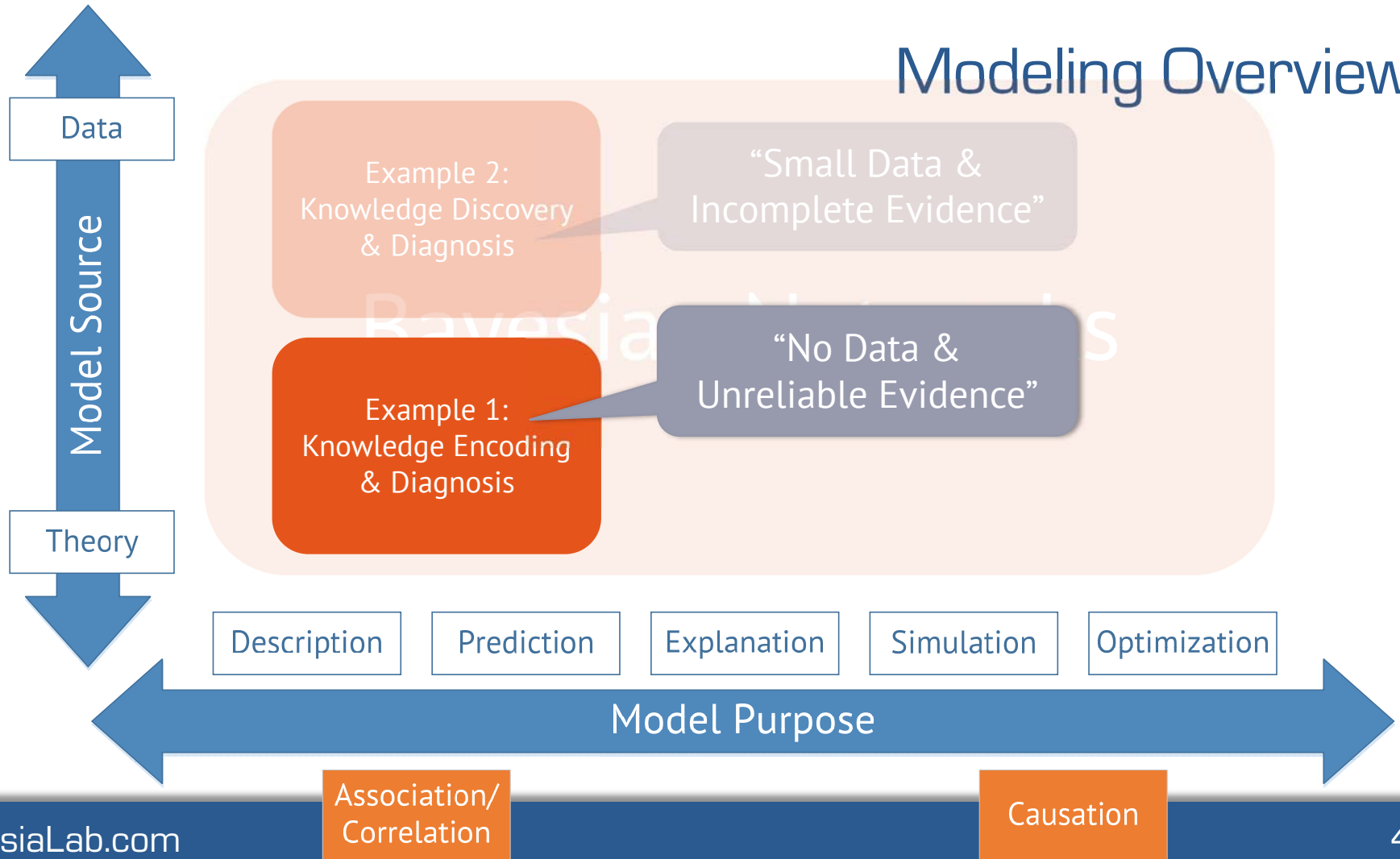
# The New Paradigm: Bayesian Networks



# Inference Engine



# Modeling Overview



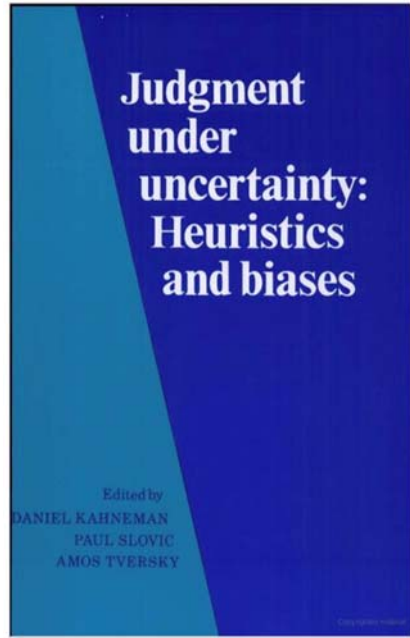


**Example 1:**  
**Probabilistic Inference**  
**Taxi Cab Example**

# Example 1

See Chapter 4

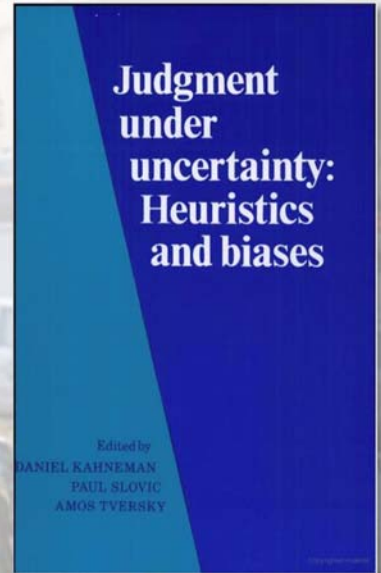
## Motivation: Human Biases in Diagnostic Reasoning



# Probabilistic Inference

## Human Reasoning Experiment (adapted from Kahneman & Tversky, 1980)

- A cab was involved in a hit-and-run accident at night.
- Two taxicab companies are operating in the city, one with yellow and one with white taxis:
  - 85% are yellow and 15% are white





# Probabilistic Inference

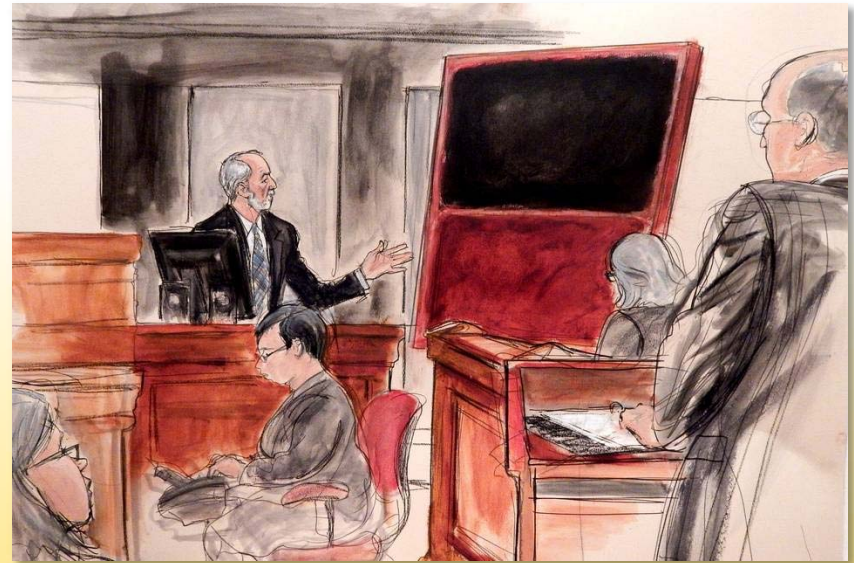
- A witness identified the taxi involved in the accident as white...

1  
1  
/  
3  
/  
2  
0  
1  
6

# Probabilistic Inference

## At the Trial

- An expert witness explains that human vision has an 80% sensitivity in terms of distinguishing between white and yellow given light conditions at the time of the accident.
- What is the probability that the taxi was actually white?



# Probabilistic Inference

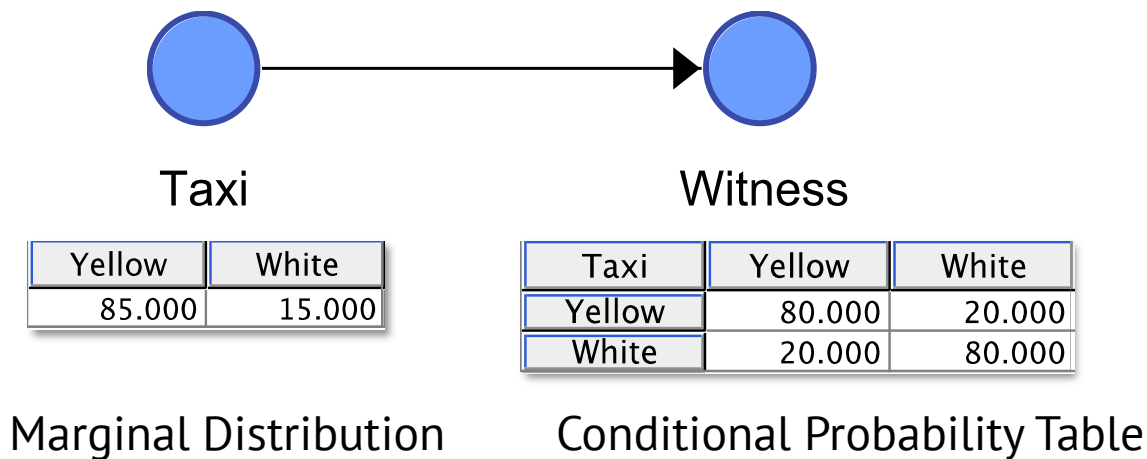
- We need to perform diagnostic probabilistic inference, i.e. from effect to cause, to answer this question.
- The Bayes Rule allows us to compute the probability:

$$P(X | Y) = \frac{P(Y | X)P(X)}{P(Y)}$$

$$P(\text{Taxi} = \text{white} | \text{Witness} = \text{white}) = \frac{P(\text{Witness} = \text{white} | \text{Taxi} = \text{white})P(\text{Taxi} = \text{white})}{P(\text{Witness} = \text{white})} =$$
$$\frac{P(\text{Witness} = \text{white} | \text{Taxi} = \text{white})P(\text{Taxi} = \text{white})}{P(\text{Witness} = \text{white} | \text{Taxi} = \text{white})P(\text{Taxi} = \text{white}) + P(\text{Witness} = \text{white} | \text{Taxi} = \text{yellow})P(\text{Taxi} = \text{yellow})}$$

# Probabilistic Inference

Representing our domain knowledge in the form of a simple Bayesian network



# Probabilistic Inference

Carrying out inference based on observing evidence

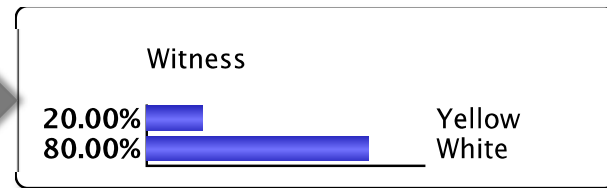
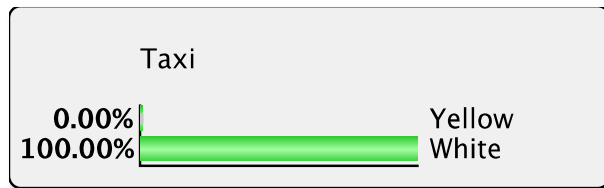


Taxi

Witness

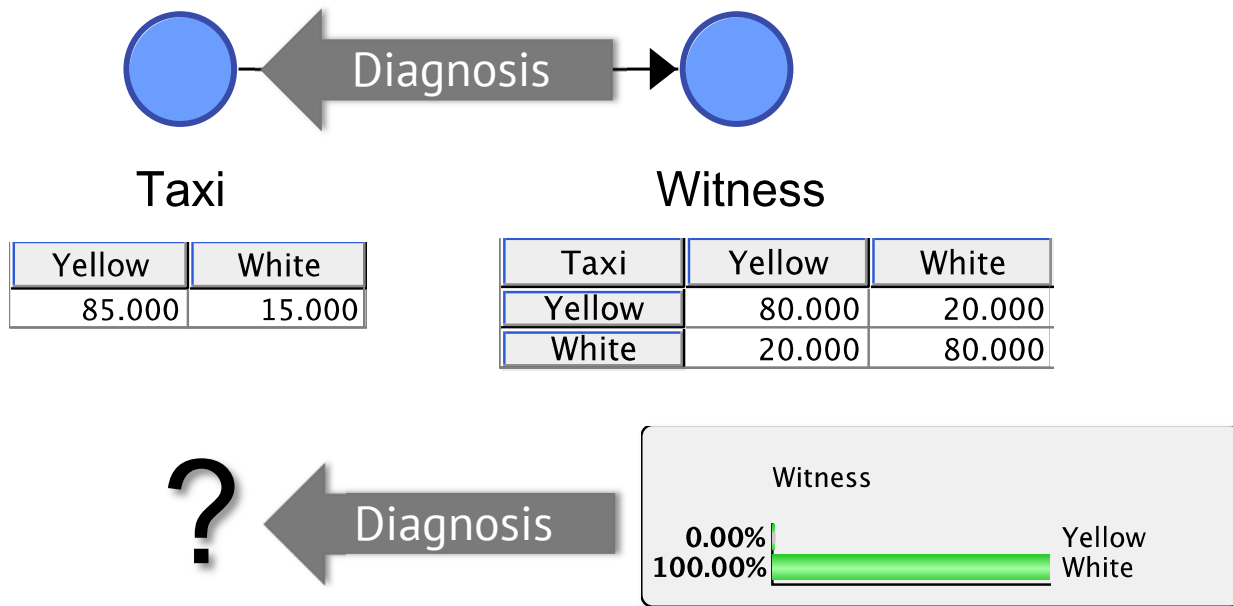
Yellow	White
85.000	15.000

Taxi	Yellow	White
Yellow	80.000	20.000
White	20.000	80.000



# Probabilistic Inference

Carrying out inference based on observing evidence



# Probabilistic Inference

Carrying out inference based on observing evidence

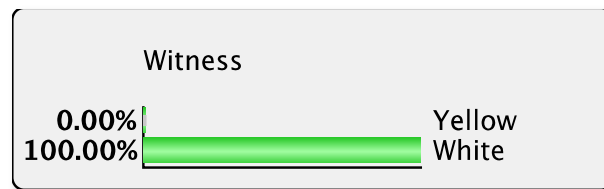
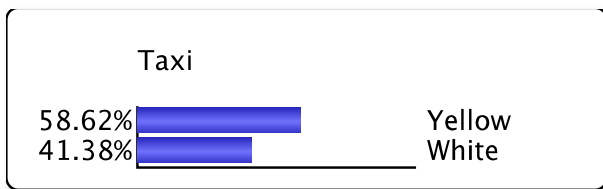


Taxi

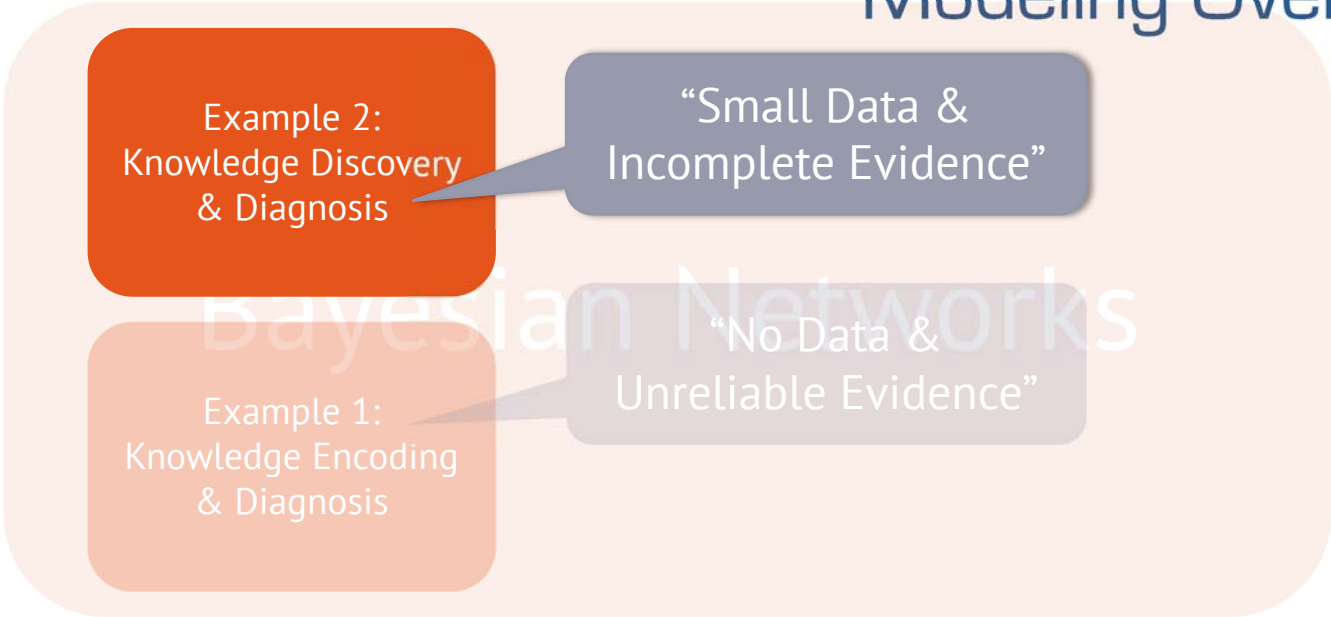
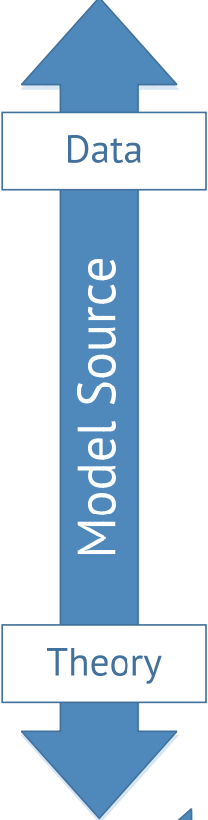
Yellow	White
85.000	15.000

Witness

Taxi	Yellow	White
Yellow	80.000	20.000
White	20.000	80.000



# Modeling Overview



Association/  
Correlation

Causation





# Example 2: Breast Cancer Diagnostics

Supervised Learning

See Chapter 6

# Breast Cancer Diagnostics

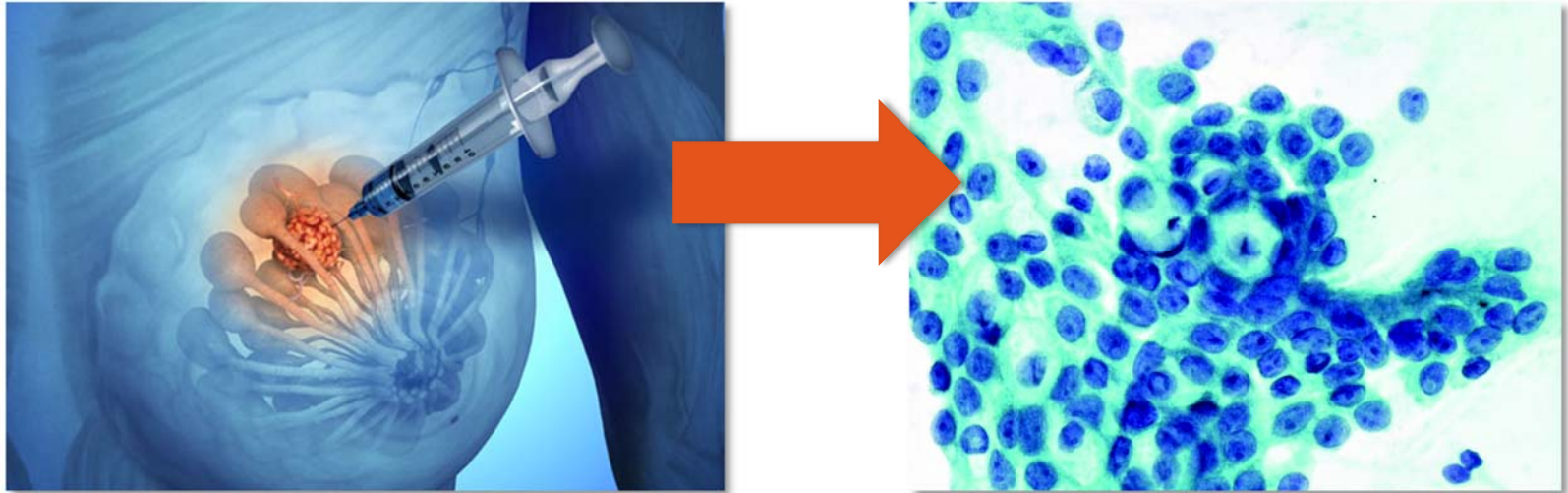
## Background for Original Study (Wolberg et al.)

- Challenge in Breast Cancer Diagnostics:
  - Mammography lacks sensitivity (i.e. true positive rate): 68% to 79%;
  - Surgical biopsy has high sensitivity (>98%), but invasive, time-consuming and costly;

# Breast Cancer Diagnostics

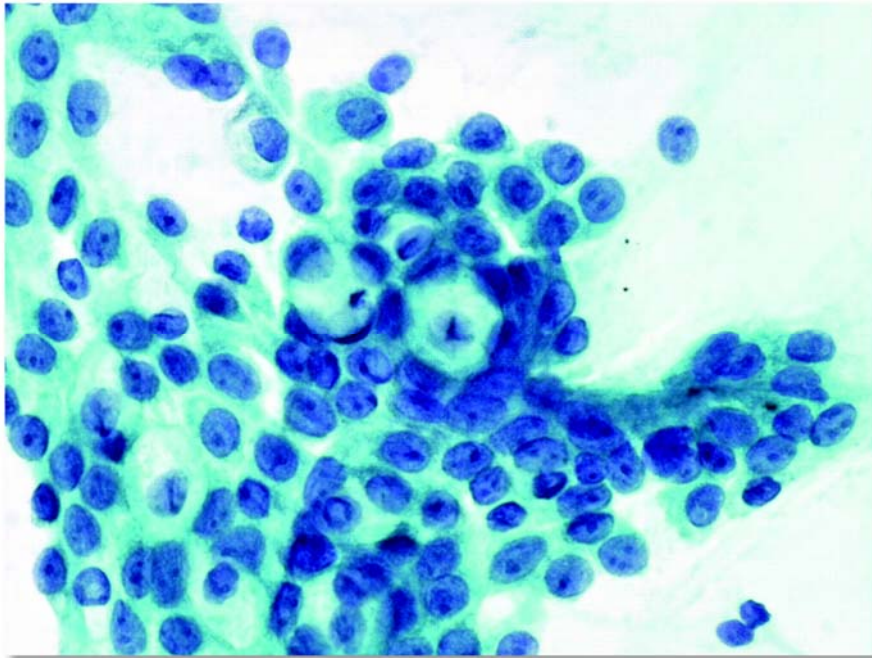
## Image Analysis of Fine Needle Aspirates

- Sensitivity of Fine Needle Aspiration with visual interpretation varies widely (65% to 98%)



# Breast Cancer Diagnostics

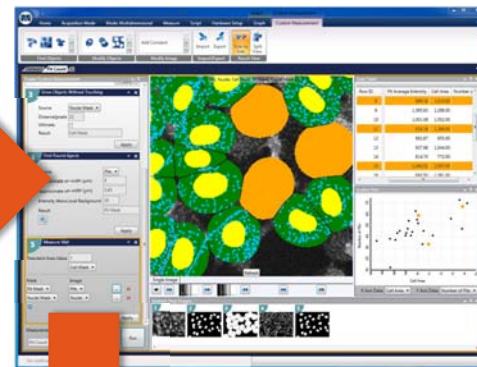
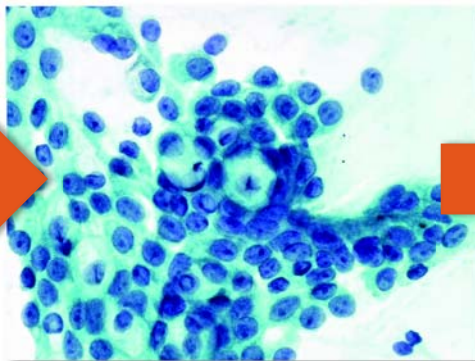
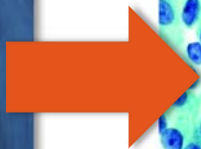
## Image Analysis of Fine Needle Aspirates



## Image Attributes

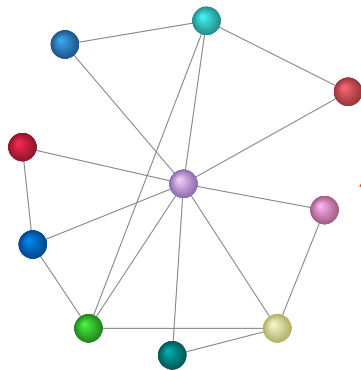
- Clump Thickness
- Uniformity of Cell Size
- Uniformity of Cell Shape
- Marginal Adhesion
- Single Epithelial Cell Size
- Bare Nuclei
- Bland Chromatin
- Normal Nucleoli
- Mitoses

# Overview



Wisconsin Breast Cancer Database

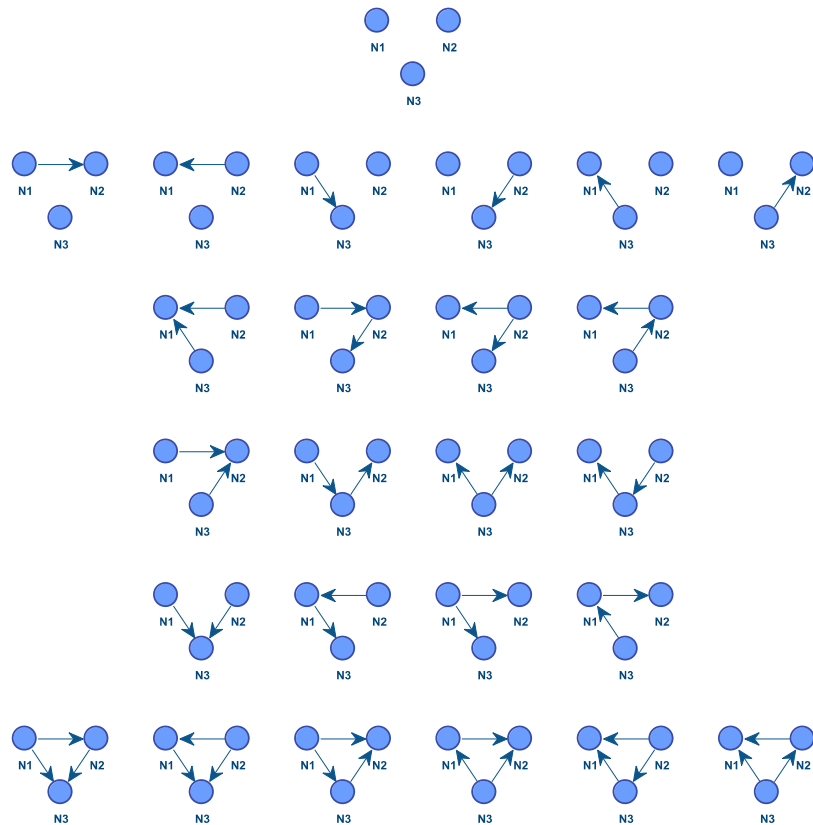
Sample Code number	Clump Thickness	Uniformity of Cell Size	Uniformity of Cell Shape	Marginal Adhesion	Single Epithelial Cell Size	Bare Nuclei	Bland Chromatin	Normal Nucleoli	Mitoses	Class
1000025	5	1	1	1	2	1	3	1	1	2
1002945	5	4	4	5	7	10	3	2	1	2
1015425	1	1	1	1	2	2	3	1	1	2
1016277	8	8	8	1	3	4	3	7	1	2
1017023	4	1	1	3	2	1	3	1	1	2
1017122	8	10	10	8	7	10	9	7	1	4
1018099	1	1	1	1	2	10	3	1	1	2
1018561	2	1	2	1	2	1	3	1	1	2
1033078	2	1	1	1	2	1	1	1	5	2



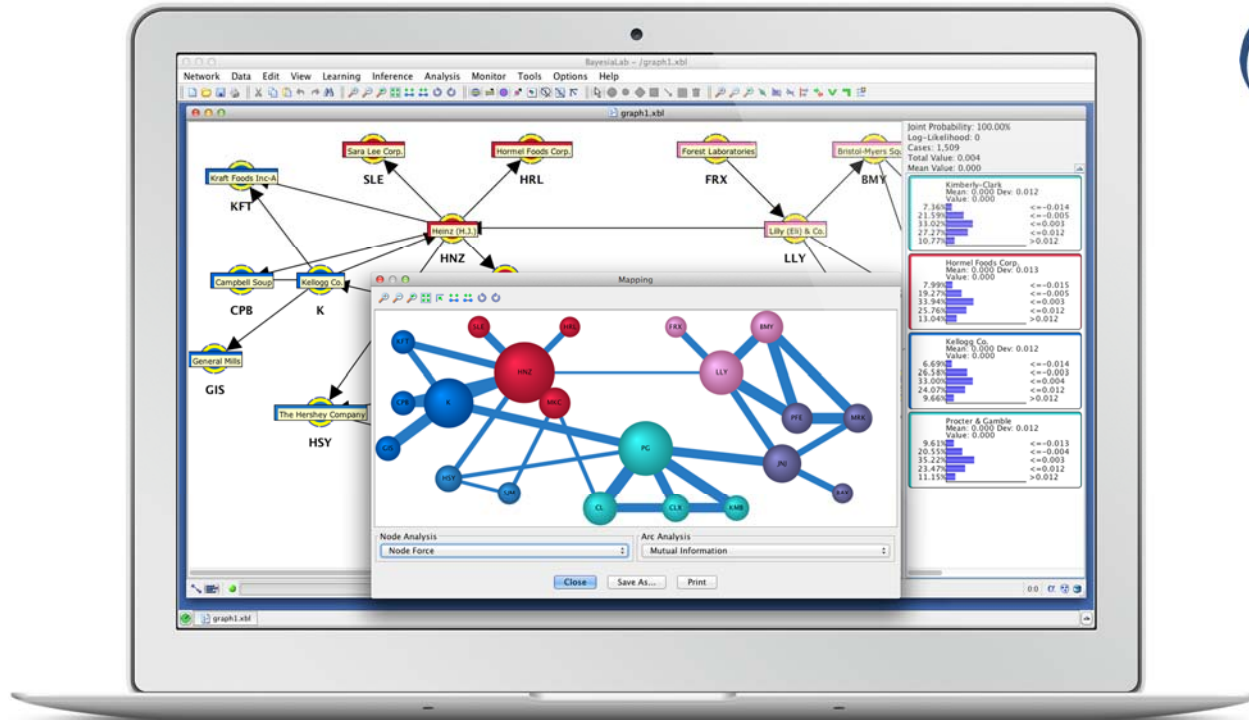
# Challenges

## Number of Possible Networks

- 2 Nodes: 3
- **3 Nodes: 25**
- 4 Nodes: 543
- 5 Nodes: 29,281
- 6 Nodes:  $3.8 \times 10^6$
- 7 Nodes:  $1.1 \times 10^9$
- 8 Nodes:  $7.8 \times 10^{11}$
- 9 Nodes:  $1.2 \times 10^{15}$
- 10 Nodes:  $4.2 \times 10^{18}$
- 11 Nodes:  $3.2 \times 10^{22}$
- 12 Nodes:  $5.2 \times 10^{26}$



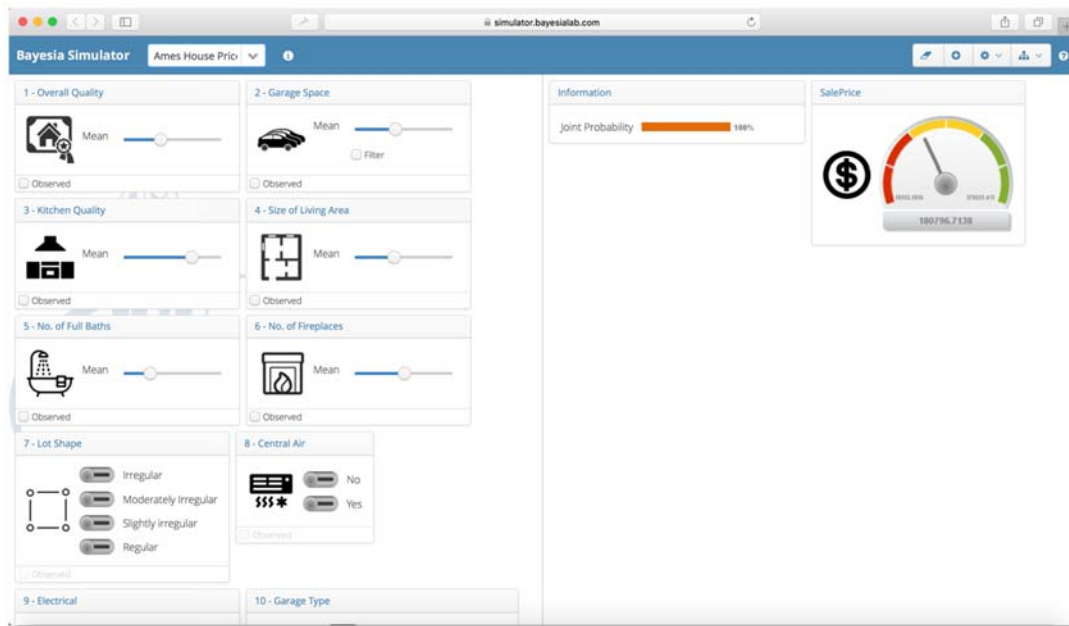
# The BayesiaLab Software Platform



- learning
- editing
- inference
- analysis
- simulation
- optimization
- publication

# Breast Cancer Diagnostics

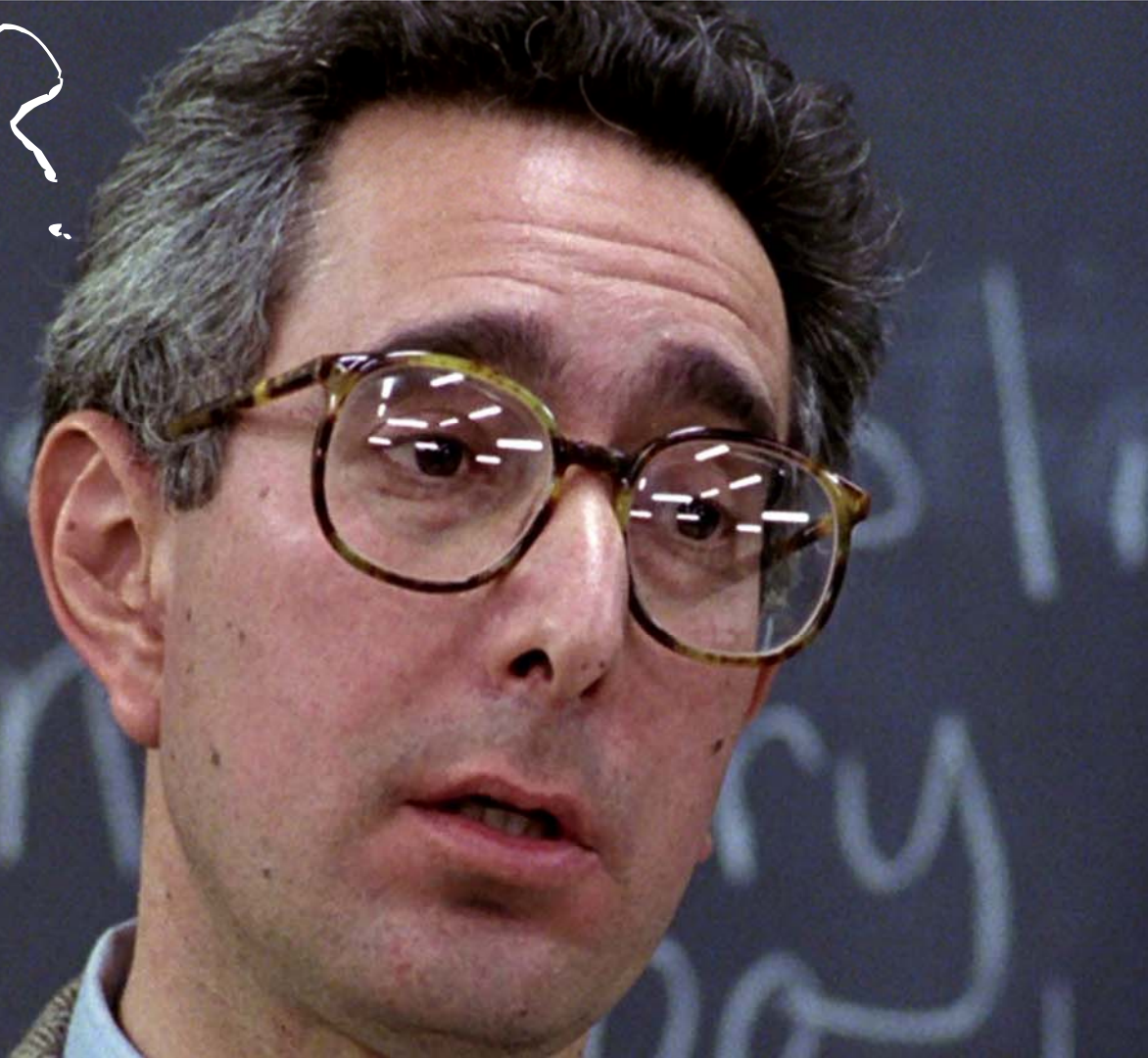
## BayesiaLab WebSimulator





QUESTIONS?

ANYONE?



# Bayesian Networks & BayesiaLab

## A Practical Introduction for Researchers

- Free download:  
[www.bayesia.com/book](http://www.bayesia.com/book)
- Hardcopy available on Amazon:  
<http://amzn.com/0996533303>



# Thank You!



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